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COMMENT ON "DYNAMIC RESPONSE OF A STRUCTURAL PANEL BY BOLOTIN'S METHOD"[1]

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In the above paper[1], the authors suggest that, for analytical purposes, the problem of the vibrational response of a rectangular box having its top face open and its base rigidly clamped can be treated by considering the response of the individual constituent panels, assuming the common edges as elastically supported. Frequency equations for such plates are derived using the Bolotin edge effect method and solved for the particular case where the elastic supports are taken to be infinitely stiff (i.e. clamped), thus the plate is considered as completely decoupled from the adjacent plates. The results so obtained are compared with experimental values for a particular box.

It is the contention of the present writer that the individual treatment of the constituent panels is not a justifiable approach to the original problem as it can lead both to relatively large errors in predicted frequencies and to the possible omission of a number of modes of vibration. It is also his belief that the free edge boundary conditions have not been treated correctly in the above paper which, unfortunately, causes certain expressions to be in error and hence the analytically derived numerical results presented to be incorrect.

Considering the latter point, regarding the boundary conditions, Ueng and Nickels correctly state that a free edge cannot support a shear force or bending moment but incorrectly interpret this as implying that, on the free edge at $x = a$,

$$W_{2,xx}(a,y) = 0 \tag{23}$$†

$$W_{2,xxx}(a,y) = 0. \tag{24}$$

The writer believes the correct interpretation to be

$$M_x = \pm D\{W_{2,xx} + \nu W_{2,yy}\} = 0$$

and

$$Q_x = \pm D\{W_{2,xxx} + (2-\nu) W_{2,xyy}\} = 0,$$

the initial sign depending upon the chosen sign convention.

Equations (27)–(29) are consequently believed to be in error and frequency equation (29) should read

$$\tan(k_1 a) = (1 + r_1\alpha/k_1)/(\alpha - r_1/k_1), \tag{29'}$$

where

$$r_1 = -(k_1^2 + 2k_2^2)^{1/2}$$

†Equation numbers from original paper.

and

$$\alpha = \frac{\{-r_1^2 + \nu k_2^2\}\{k_1^3 + (2-\nu)k_2^2 k_1\}}{\{k_1^2 + \nu k_2^2\}\{-r_1^3 + (2-\nu)k_2^2 r_1\}}.$$

The first five frequencies for modes symmetrical about the centreline $y = b/2$ were computed by the writer using eqns (29′) and (39) for the $12 \times 6 \times 0.06$ in. ($305 \times 152 \times 1.5$ mm) plate considered by Ueng and Nickels and are presented in Table 1 together with Ueng's and Nickels' results and values obtained using beam mode shapes in Rayleigh's method[2] for confirmation purposes. It may be seen that there is close agreement between the Rayleigh results and those from eqns (29′) and (39), helping to confirm the validity of the latter, and that there are some discrepancies between the present results and those of Ueng and Nickels. It may also be pointed out that, while Rayleigh's method gives upper bounds, the edge effect method is believed to yield lower bounds for single plates[3] (although not necessarily for plate systems[4]), a trend not contradicted by the results of Table 1.

Returning to the original problem of the box structure, the present writer has used the Bolotin edge effect method to derive equations for the system using the approach described in Ref. [4]. This approach treats the system as a whole and thus accounts for coupling between adjacent plates. Since the box is symmetrical about two planes, $y = b/2$ and $z = c/2$ (see Fig. 1) it is able to vibrate in four families of modes:

Type 1. Modes symmetrical about both $y = b/2$ and $z = c/2$;
Type 2. Modes symmetrical about $y = b/2$ and antisymmetrical about $z = c/2$;
Type 3. Modes antisymmetrical about $y = b/2$ and symmetrical about $z = c/2$;
Type 4. Modes antisymmetrical about both $y = b/2$ and $z = c/2$.

Although the results in Ueng's and Nickels' paper are for modes of Type 1 only, the relevant equations for all four types are given here for completeness. If use is made of this symmetry and antisymmetry, the problem reduces to that of the analysis of a two-plate system as shown in Fig. 2, where the edges $y = b/2$ and $z = c/2$ are treated either as "sliding" (an axis of symmetry—zero shear force and zero slope) or simply supported (an axis of antisymmetry—nodal line). It is now convenient to associate the wavelength parameters k_1 and k_2 with directions x and y respectively in plate 1 and k_4 and k_3 with directions x and z in plate 2. For the two-plate system, four equations result in parameters k_1 to k_4: one from consideration of the x direction in plate 1, which is identical to eqn (29′); one from consideration of the x direction in plate 2, which is found to be identical in form to eqn (29′) but with the subscripts 1 and 2 replaced by 4 and 3 respectively; one from the frequency relationship

$$\omega^2 m/D = (k_1^2 + k_2^2)^2 = (k_4^2 + k_3^2)^2;$$

and one from consideration of the y and z directions in plates 1 and 2 respectively, with the appropriate boundary conditions on $y = b/2$ and $z = c/2$, zero transverse deflection at the common edge $y = 0$, $z = 0$ and continuity of slope and bending moment on that edge. There are four versions of this last equation depending upon the symmetry or antisymmetry of the mode:

Mode type 1.

$$\beta_3 \cos(k_2 b/2) + \beta_2 \cos(k_3 c/2) = 0.$$

Table 1. Natural frequencies for the C–C–F–C plate in Hz

	Ueng and Nickels[1]	Equations (29′) and (39)	Rayleigh's method[2]
f_1	360	362	368
f_2	440	417	425
f_3	503	544	548
f_4	761	754	758
f_5	1023	1050	1056

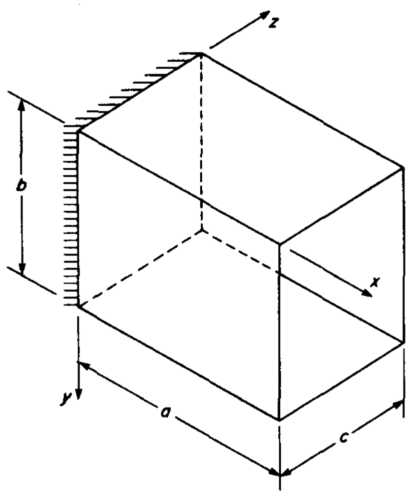

Fig. 1. Cantilevered box structure.

Mode type 2.

$$\gamma_3 \cos(k_2 b/2) - \beta_2 \sin(k_3 c/2) = 0;$$

Mode type 3.

$$\beta_3 \sin(k_2 b/2) - \gamma_2 \cos(k_3 c/2) = 0;$$

Mode type 4.

$$\gamma_3 \sin(k_2 b/2) + \gamma_2 \sin(k_3 b/2) = 0$$

where

$$\beta_2 = k_2 \sin(k_2 b/2) - r_2 \cos(k_2 b/2)$$

$$\beta_3 = k_3 \sin(k_3 c/2) - r_3 \cos(k_3 c/2)$$

$$\gamma_2 = k_2 \cos(k_2 b/2) + r_2 \sin(k_2 b/2)$$

$$\gamma_3 = k_3 \cos(k_3 c/2) + r_3 \sin(k_3 c/2)$$

and

$$r_3 = -(k_3^2 + 2k_4^2)^{1/2}, \qquad r_2 = -(k_2^2 + 2k_1^2)^{1/2}.$$

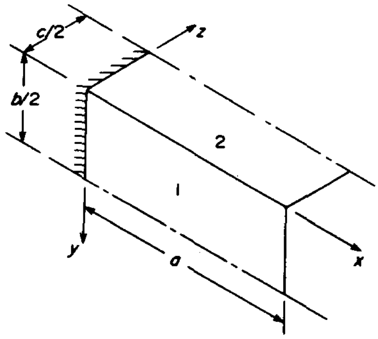

Fig. 2. Equivalent two-plate system.

Substitution for k_4 from the frequency relationship stated earlier leads to three simultaneous transcendental equations in parameters k_1, k_2 and k_3, the solution of which allows the determination of the frequency parameter $\omega^2 m/D$. (It should be mentioned that such transcendental equations admit many solutions which are not meaningful[4, 5] but these may be identified by inspection of the wavelength parameters defining the deflection of adjacent plates in the direction parallel to a common edge—k_1 and k_4 in this case. For the solutions to be meaningful, these parameters must be of similar order.)

The writer has solved the set of equations for modes of Type 1 for comparison with Ueng's and Nickels' experimental results and the single panel (C–C–F–C) representation. He has also solved the single panel problem assuming the common edges to be clamped (cl) and simply supported (SS) using both Bolotin's method and Rayleigh (for confirmation). Table 2 shows the comparisons and it may be seen that the results from the full analysis by the Bolotin method agree very well with the reported experimental frequencies. They suggest, however that even if only modes of Type 1 (fully symmetrical) are considered, the C–C–F–C panel approximation (identified by cl in columns 1, 4–5) does not lead to a close approximation of the lower modes of the box and omits fully half the existing modes. If, in the approximate analysis, the elastic restraints are relaxed completely in a rotational sense, the common edges thus becoming simply supported, then additional values will be obtained (identified by SS in columns 4 and 5) and it appears that the correct number of modes will have been found. The applicability of the frequencies so obtained, however, is questionable. In fact, the "clamped" and "simply supported" results for the single panel are the same as those for the fully symmetrical modes of vibration of a box of dimensions 12 × 6 × 6 in. (305 × 152 × 152 mm). Clearly, a similar analysis could have been performed on the 12 × 8 in. (305 × 203 mm) panel, yielding a whole new set of frequencies of doubtful applicability.

In conclusion, then, the writer believes that the Bolotin edge effect method can be used for the determination of the natural frequencies of box-like structures, although the application to five or six sided boxes involves some difficulties due to the existence of modes having nodal lines not easily representable by lines parallel to the plate edges[4–6]. However, even for the four plate system, he believes that confidence in the results can only be achieved if the full structure is analyzed, allowing for coupling between adjacent panels, and that the treatment of isolated individual panels is not valid.

Table 2. Natural frequencies in Hz for the 12 × 8 × 6 in. (305 × 203 × 152 mm) box of Ueng and Nickels

Ueng and Nickels		Present analysis (Bolotin)		Rayleigh [2]
Calculated	Measured	Box	Single 12 × 6 in. panel	Single panel
—	—	119	161 (SS)	171 (SS)
—	—	201	253 (SS)	259 (SS)
360 (cl)	290	292	361 (cl)	368 (cl)
440 (cl)	360	354	417 (cl)	425 (cl)
—	—	359	414 (SS)	417 (SS)
503 (cl)	510	490	544 (cl)	548 (cl)
—	—	594	654 (SS)	654 (SS)
761 (cl)	750	710	754 (cl)	758 (cl)
—	—	910	971 (SS)	972 (SS)
1023 (cl)	1090	1015	1050 (cl)	1056 (cl)

REFERENCES

1. C. E. S. Ueng and R. C. Nickels, Jr., Dynamic response of a structural panel by Bolotin's method. *Int. J. Solids Structures* **14**, 571–578 (1978).
2. G. B. Warburton, Vibration of rectangular plates. *Proc. Inst. Mech. Engrs* **168**(12), 371–384 (1954).
3. V. V. Bolotin, An asymptotic method for the study of the problem of eigenvalues for rectangular regions. In *Problems of Continuum Mechanics* (Volume dedicated to N. I. Muskhelishvili), pp. 56–68. SIAM, New York (1961).
4. S. M. Dickinson and G. B. Warburton, Natural frequencies of plate systems using the edge effect method. *J. Mech. Engng Sci.* **9**(4), 318–324 (1967).
5. S. M. Dickinson, Flexural vibration of rectangular plate systems. Ph.D. Thesis, Mechanical Engineering Department, Nottingham University (1966).
6. S. M. Dickinson and G. B. Warburton, Vibration of box-type structures. *J. Mech. Engng Sci.* **9**(4), 325–335 (1967).